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ALTERNATIVE TREATMENT OF DATA ENVELOPMENT ANALYSIS MODEL WITH CONGESTION MEASUREMENT

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Abstract

Data envelopment analysis (DEA) is a mathematical programming approach for measuring the relative efficiencies within a group of decision making units (DMUs). An important outcome of such an analysis is a set of values for dual variables which assist the measuring of congestion. This paper decomposes the normalizing equation in the original DEA model to evaluate the congestion.

Keywords: Data envelopment analysis; Congestion; Efficiency

Introduction

The data envelopment analysis (DEA) method, first proposed by Charnes et al. (1978), is widely known as an evaluation technique for efficiency measure within a group of decision making units (DMUs) based on multiple inputs and outputs. The efficiency of a DMU within the DEA frame is defined as the ratio of multiple weighted outputs to multiple weighted inputs. Under the DEA restriction that no DMU has more than 100% efficiency, the weights are chosen to show that a specific

DMU is as efficient as possible. If the efficiency score of a DMU calculated by the DEA model is of one, then the DMU is said to be relatively efficient. Otherwise; it is relatively inefficient.

The efficiency of DEA concept is a weak efficiency. The weak efficiency is also referred to as free disposal in economic literature. This concept of free disposal is further refined by Färe et al. (1985) in a way that distinguishes between "strong disposal" and "weak disposal." If the efficiency that conforms to the condition of strong disposal is smaller than conforms to that of weak disposal, congestion is identified. To measure and evaluate congestion, Färe et al. (1985) proposed an operational approach (namely FGL approach) and then the treatment of congestion within the DEA context has received considerable attention in literature (e.g. Cherchye et al., 2001; Cooper et al., 2000; Cooper et al., 2001; Färe and Grosskopf, 2000). However, Cooper et al. (2001) found that the FGL approach was not totally correct.

This study aims to advance the work of Färe et al. (1985) on congestion measure and to offer another choice of improvement targets for inefficient DMUs. This paper is organized as follows. Section 2 presents a brief introduction of the measure of congestion. In Section 3, we modify the CCR model to advance the congestion measure. Our conclusions are offered in Section 4.

The measure of congestion

Suppose there are n DMUs with s outputs and m inputs to be evaluated, and denote y_{ik} as the volume of output i and x_{rk} as that of input r of DMU$_k$. The efficiency measure for DMU$_j$ is a solution from the following linear programming (LP) model referred to as the CCR model (Charnes *et al.*, 1978).

$$\text{Max} \quad \sum_{i=1}^{s} u_i y_{ij} \tag{1a}$$

$$\text{s.t} \quad \sum_{r=1}^{m} v_r x_{rj} = 1 \tag{1b}$$

$$\sum_{i=1}^{s} u_i y_{ik} - \sum_{r=1}^{m} v_r x_{rk} \leq 0,\ k = 1,2,\dots,n \tag{1c}$$

$$u_i, v_r \geq \varepsilon > 0,\ i = 1,2,\dots,s,\ \ r = 1,2,\dots,m. \tag{1d}$$

Where u's and v's are decision variables associated with outputs and inputs, respectively, and ε is a positive non-Archimedean infinitesimal. Constraint (1b) is referred to as the normalizing equation (Dyson *et al.*, 2001). Model (1) is an input-oriented CCR model (hereafter based on this model) and it allows each DMU to effectively select best weights that maximize the weighted output, but at the same time the constraint set prevents the efficiencies of the *n* DMUs calculated with these weights from exceeding a value of one.

The dual of Model (1) is given by

$$\text{Min } g_j = \theta - \varepsilon\left(\sum_{i=1}^{s} S_i^{+} + \sum_{r=1}^{m} S_r^{-}\right) \tag{2a}$$

$$\text{s.t } \sum_{k=1}^{n} \lambda_k x_{rk} - \theta x_{rj} - S_r^{-} = 0, \quad r = 1,2,\ldots,m \tag{2b}$$

$$\sum_{k=1}^{n} \lambda_k y_{ik} - S_i^{+} = y_{ij}, \quad i = 1,2,\ldots,s \tag{2c}$$

$$\lambda_k, S_r^{-}, S_i^{+} \geq 0, \quad k = 1,2,\ldots,n, \quad i = 1,2,\ldots,s, \quad r = 1,2,\ldots,m \tag{2d}$$

$$\theta \text{ unrestricted.} \tag{2e}$$

Where θ and λ_k are dual variables and a value of $\theta^* < 1$ implies that DMU$_j$ is inefficient. For inefficient DMU$_j$, the improvement target thus identifies the wasted amount of input *r* is $(1-\theta_j^*)x_{rj}$, $r = 1,2,\ldots,m$. This implies that each input has the same adjustment proportions.

The FGL approach proceeds in two stages. In stage one, Model (2) is employed and conforms to the condition of strong disposal. A model with which exhibits weak (input) disposal is shown as

$$\beta_j^* = \text{Min } \beta \tag{3a}$$

$$\text{s.t. } \sum_{k=1}^{n} \lambda_k x_{rk} - \beta x_{rj} = 0, \quad r = 1,2,\ldots,m \tag{3b}$$

$$\sum_{k=1}^{n} \lambda_k y_{ik} - S_i^{+} = y_{ij}, \quad i = 1,2,\ldots,s \tag{3c}$$

$$\beta, \lambda_k, S_i^{+} \geq 0, \quad k = 1,2,\ldots,n, \quad i = 1,2,\ldots,s, \quad r = 1,2,\ldots,m. \tag{3d}$$

Comparing Model (2) and Model (3), Model (3) is more restricted than Model (2) , and hence we obtain the relationships $0 \le g_j^* \le \beta_j^* \le 1$, i.e.

$$0 \le \frac{g_j^*}{\beta_j^*} \le 1. \tag{4}$$

Färe et al. (1985) utilized this property to develop a measure of congestion referred to as the second stage. The congestion is identified as present if and only if $\frac{g_j^*}{\beta_j^*} < 1$ and as not present if and only if $\frac{g_j^*}{\beta_j^*} = 1$ in the performance of DMU$_j$. An example was illustrated by Färe et al (see Table 1) to demonstrate the FGL approach. However, Cooper et al. (2001) utilized the data to Model (2) and Model (3), and found that the obtained efficiency scores of DMU$_6$ are $g_6^* = 0.8$ and $\beta_6^* = 0.86$ respectively, so that $\frac{g_6^*}{\beta_6^*} = 0.93 < 1$. According to this result, Cooper et al. (2001) concluded that the FGL approach was not correct. In fact, as can be seen in Figure 1, DMU$_6$ is technical inefficiency rather than congestion. Therefore, we need to modify the FGL approach for the measure of congestion.

Modification of the CCR model

To overcome the shortcoming of FGL approach, this study modifies the CCR model by decomposing (1b) into m components, i.e. $v_1 x_{1j} \le \alpha_1$, $v_2 x_{2j} \le \alpha_2, \ldots,$ $v_m x_{mj} \le \alpha_m$, and joins them into Model (2) shown as Model (5).

$$\text{Max} \quad e_j = \sum_{i=1}^{s} u_i y_{ij} \tag{5a}$$

$$\text{s.t} \qquad v_r x_{rj} \le \alpha_r, \quad r = 1,2,\ldots,m \tag{5b}$$

Table 1. Congestion example[a]

DMU	Output	Inputs		Efficiency
	y	x_1	x_2	
1	2	1	2	1.0
2	2	2	2	0.75
3	2	2	1	1.0
4	2	1	3	1.0
5	2	1	4	1.0
6	2	3	1.25	0.8
7	2	4	1.25	0.8

[a]Source: Färe et al. (1985, p.76)

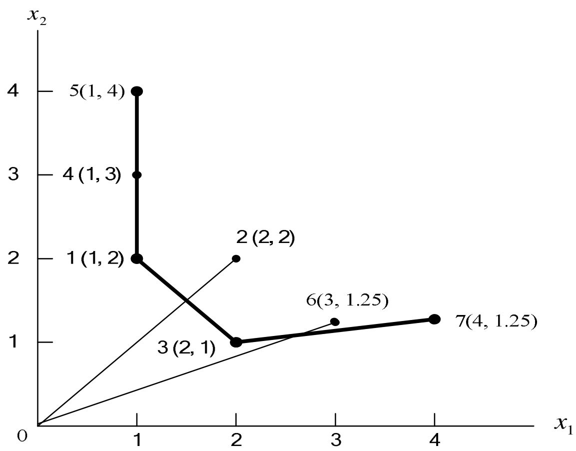

Figure 1. Efficiency points and frontier

$$\sum_{i=1}^{s} u_i y_{ik} - \sum_{r=1}^{m} v_r x_{rk} \le 0,\ k = 1,2,\ldots,n \tag{5c}$$

$$u_i, v_r \ge \varepsilon > 0\ ,\ \alpha_r \ge 0,\ i = 1,2\ldots,s\ ,\ \ r = 1,2,\ldots,m\ , \tag{5d}$$

where $\sum_{r=1}^{m} \alpha_r = 1$. The dual problem of Model (5) is shown as Model (6).

$$\text{Min}\quad h_j = \sum_{r=1}^{m} \alpha_r \theta_r - \varepsilon\left(\sum_{i=1}^{s} S_i^+ + \sum_{r=1}^{m} S_r^-\right) \tag{6a}$$

$$\text{s.t}\quad \theta_r x_{rj} - \sum_{k=1}^{n} \lambda_k x_{rk} - S_r^- = 0\ ,\ \ r = 1,2,\ldots,m\ , \tag{6b}$$

$$\sum_{k=1}^{n} \lambda_k y_{ik} \qquad - S_i^+ = y_{ij}, \; i = 1,2,\ldots,s, \tag{6c}$$

$$\theta_r, \lambda_k, S_r^-, S_i^+ \geq 0, \; k = 1,2,\ldots,n, \; i = 1,2,\ldots,s, \; r = 1,2,\ldots,m. \tag{6d}$$

To obtain the efficiency score of DMU$_j$ by Model (5) or Model (6), we need to specify the value of α_r, and the following procedure is proposed.

Step 1. Run the CCR model to obtain the optimal weights of inputs, i.e. v_r^*, $r = 1,2,\ldots,m$.

Step 2. Calculate the ceiling ratio of input *r* by letting $\alpha_r = v_r^* x_{rj}$.

In a way similar to the FGL approach, we utilize $\theta_r x_{rj} - \sum_{k=1}^{n} \lambda_k x_{rk} = 0$ to replace Equation (6b) to exhibit weak (input) disposal for DMU$_j$, and Model (6) is converted to Model (7).

$$\text{Min} \quad f_j = \sum_{r=1}^{m} \alpha_r \theta_r - \varepsilon \sum_{i=1}^{s} S_i^+ \tag{7a}$$

$$\text{s.t} \quad \theta_r x_{rj} - \sum_{k=1}^{n} \lambda_k x_{rk} = 0, \; r = 1,2,\ldots,m \tag{7b}$$

$$\sum_{k=1}^{n} \lambda_k y_{ik} \qquad - S_i^+ = y_{ij}, \; i = 1,2,\ldots,s \tag{7c}$$

$$\theta_r, \lambda_k, S_i^+ \geq 0, \; k = 1,2,\ldots,n, \; i = 1,2,\ldots,s, \; r = 1,2,\ldots,m. \tag{7d}$$

By utilizing the example data to Model (7), the obtained efficiency score of DMU$_6$ is $f_6^* = 0.8$, and hence $\frac{g_6^*}{f_6^*} = 1$. This result supports the argument of Cooper et al. (2001) that DMU$_6$ is technical inefficiency rather than congestion. The major difference between Model (3) and Model (7) is that we consider the efficiencies contribution of individual inputs to a DMU and the item $\varepsilon \sum_{i=1}^{s} S_i^+$ in the objective function of Model (7). Based on the proposed treatment of the CCR model, the FGL approach is available. Therefore, the modified DEA model can enhance the FGL method in the measuring of congestion.

Conclusion

The characteristic of DEA is that it allows DMUs to select the best weights in calculating their efficiencies. As a by-product, the dual variables provide an insight of how the DMU being evaluated can be improved as far as the efficiency score is concerned. Nevertheless, in transforming the oriented primal model into a dual problem, decision makers traditionally give a dual variable associated with the normalizing equation. This treatment means that all inputs correspond to an equivalent dual variable, although, in essence, this is not necessary. This study modifies the CCR model to provide another choice of efficiency improvement.

The proposed approach is implemented in two stages. The first stage concerns the usual evaluation of relative efficiency. If the efficiency score calculated by the CCR model is of one, then no further effort is required: the DMUs being evaluated are already Pareto efficient. The second stage proceeds only for inefficient DMUs to evaluate congestion or to search for efficiency improvement. This technique can give each factor a different dual variable instead of equivalence. The advantages of the proposed treatment of the CCR model are that it not only can provide another choice of targets of inputs/outputs for inefficient DMUs to achieve Pareto efficiency but also can enhance the FGL approach.

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